velocity; and let any body E, by being placed in the axis of the canal, hinder the paſſage of the water. It is evident, that the figure of the fore part of this body will have little influence in obſtructing the water's motion, but the whole impediment will ariſe from the ſpace taken up by the body, by which it diminiſhes the bore of the canal, and ſtraightens the paſſage of the water[a]. But proportional to the obſtruction of the water's motion, will be the force of the water upon the body E[b]. Now ſuppoſe both orifices of the canal to be cloſed, and the water in it to remain at reſt; the body E to move, ſo that the parts of the water may paſs by it with the ſame degree of velocity, as they did before; it is beyond contradiction, that the preſſure of the water upon the body, that is, the reſiſtance it gives to its motion, will remain the ſame; and therefore will have little connexion with the figure of the body[c].

25. BY a method of reaſoning drawn from the ſame fountain is determined the meaſure of reſiſtance theſe compreſſed fluids give to bodies, in reference to the proportion between the denſity of the body and that of the fluid. This ſhall be explained particularly in my comment on Sir IS. NEWTON's mathematical principles of natural philoſophy; but is not a proper ſubject to be inſiſted on farther in this place.

26. WE have now gone through all the parts of this theory. There remains nothing more, but in few words to mention the experiments, which our author has made, both

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[a] Vid. Princ. philoſ. Lib. II. Lem. 5. p. 314. | [b] Lemm. 6. | [c] Ibid. 7.

with bodies falling perpendicularly through water, and the air[a], and with pendulums[b]: all which agree with the theory. In the caſe of falling bodies, the times of their fall determined by the theory come out the ſame, as by obſervation, to a ſurprizing exactneſs; in the pendulums, the rod, by which the ball of the pendulum hangs, ſuffers reſiſtance as well as the ball, and the motion of the ball being reciprocal, it communicates ſuch a motion to the fluid, as increaſes the reſiſtance; but the deviation from the theory is no more, than what may reaſonably follow from theſe cauſes.

27. By this theory of the reſiſtance of fluids, and theſe experiments, our author decides the queſtion ſo long agitated among natural philoſophers, whether all ſpace is abſolutely full of matter. The Ariſtotelians and Carteſians both aſſert this plenitude; the Atomiſts have maintained the contrary. Our author has choſe to determine this queſtion by his theory of reſiſtance, as ſhall be explained in the following chapter.

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[a] Newt. Princ. Lib. II. prop. 40. in ſchol.

[b] Lib. II. in ſchol. poſt prop. 31.

J. Gryſon del: J. Pine Sculp.

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BOOK II.

CONCERNING THE SYSTEM of the WORLD.

Chap. I.

That the Planets move in a ſpace empty of all ſenſible matter.

I HAVE now gone through the firſt part of my deſign, and have explained, as far as the nature of my undertaking would permit, what Sir ISAAC NEWTON has delivered in general concerning the motion of bodies. It follows now to ſpeak of the diſcoveries, he has made in the ſyſtem of the world;

and to ſhew from him what cauſe keeps the heavenly bodies in their courſes. But it will be neceſſary for the uſe of ſuch, as are not skilled in aſtronomy, to premiſe a brief deſcription of the planetary ſyſtem.

2. This ſyſtem is diſpoſed in the following manner. In the middle is placed the ſun. About him ſix globes continually roll. Theſe are the primary planets; that which is neareſt to the ſun is called Mercury, the next Venus, next to this is our earth, the next beyond is Mars, after him Jupiter, and the outermoſt of all Saturn. Beſides theſe there are diſcovered in this ſyſtem ten other bodies, which move about ſome of theſe primary planets in the ſame manner, as they move round the ſun. Theſe are called ſecondary planets. The moſt conſpicuous of them is the moon, which moves round our earth; four bodies move in like manner round Jupiter; and five round Saturn. Thoſe which move about Jupiter and Saturn, are uſually called ſatellites; and cannot any of them be ſeen without a teleſcope. It is not impoſſible, but there may be more ſecondary planets, beſide theſe; though our inſtruments have not yet diſcovered any other. This diſpoſition of the planetary or ſolar ſyſtem is repreſented in fig. 89.

3. The ſame planet is not always equally diſtant from the ſun. But the middle diſtance of Mercury is between $\frac{1}{3}$ and $\frac{2}{5}$ of the diſtance of the earth from the ſun; Venus is diſtant from the ſun almoſt $\frac{3}{4}$ of the diſtance of the earth; the middle diſtance of Mars is ſomething more than

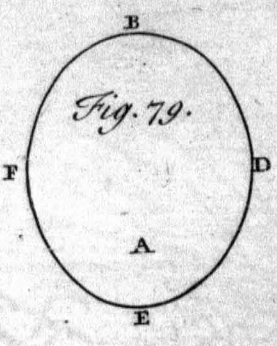

Fig. 79.

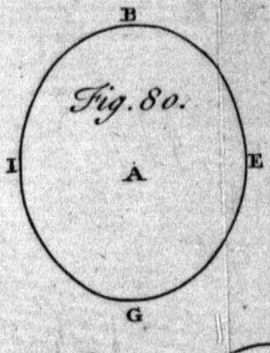

Fig. 80.

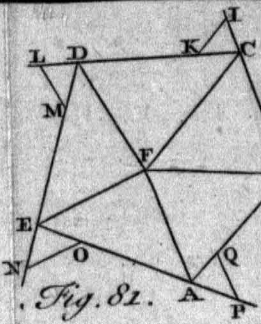

Fig. 81.

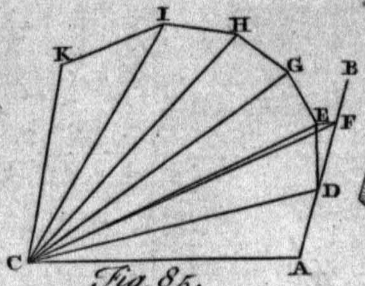

Fig. 85.

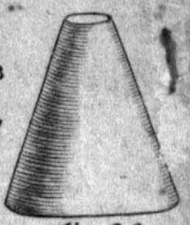

Fig. 86.

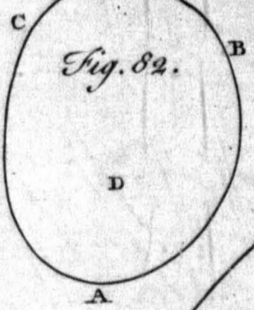

Fig. 82.

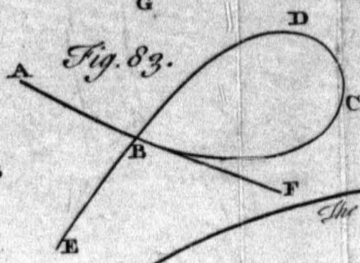

Fig. 83.

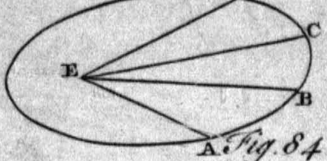

Fig. 84.

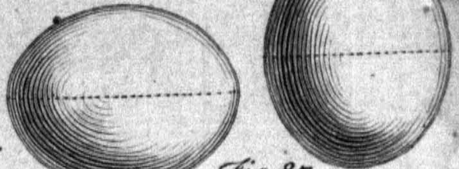

Fig. 87

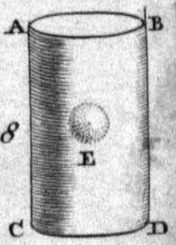

Fig. 88

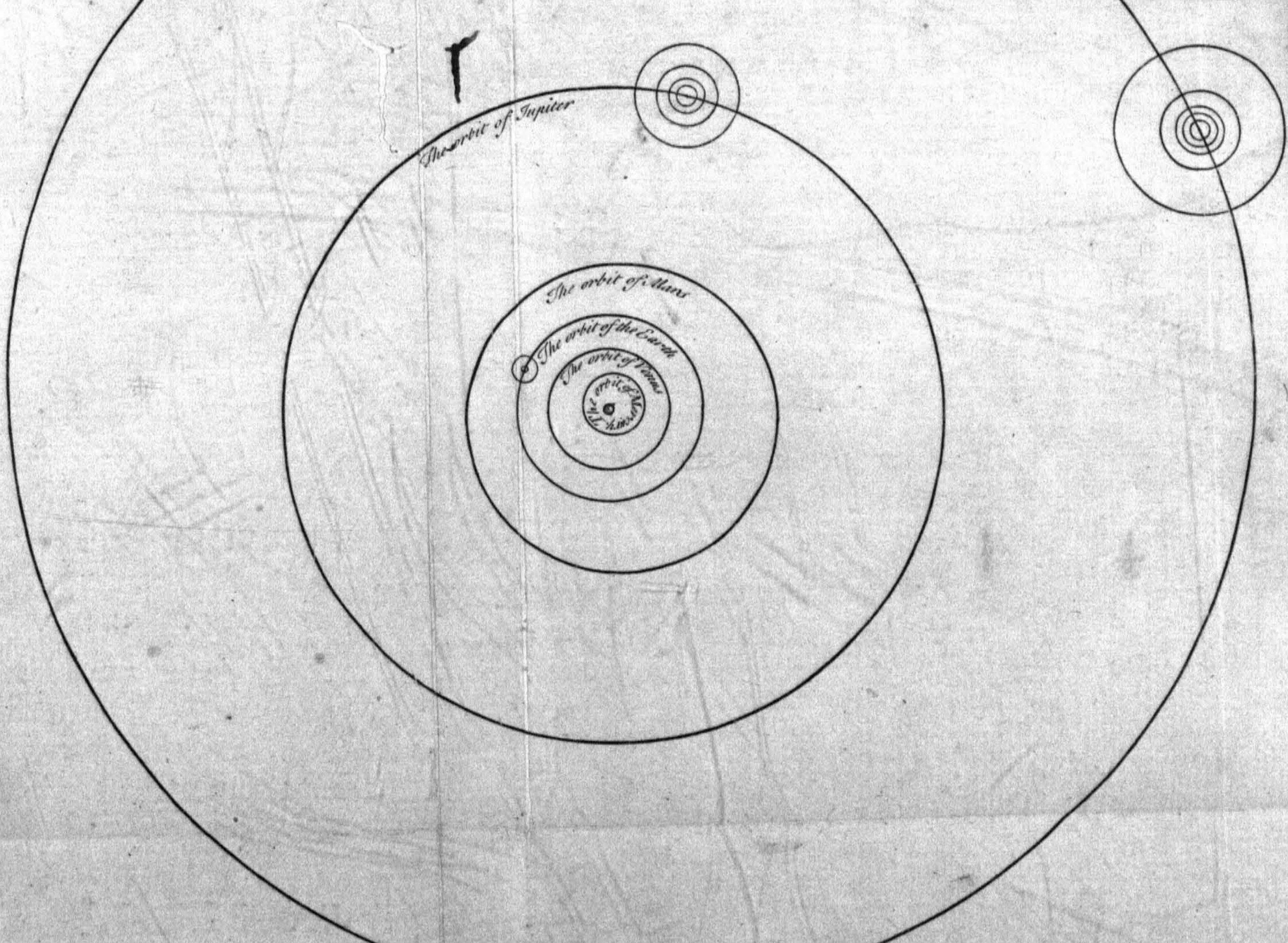

Fig. 89.

half as much again, as the diſtance of the earth; Jupiter's middle diſtance exceeds five times the diſtance of the earth, by between $\frac{1}{5}$ and $\frac{1}{6}$ part of this diſtance; Saturn's middle diſtance is ſcarce more than $9\frac{1}{2}$ times the diſtance between the earth and ſun; but the middle diſtance between the earth and ſun is about $217\frac{1}{8}$ times the ſun's ſemidiameter.

4. All theſe planets move one way, from weſt to eaſt; and of the primary planets the moſt remote is longeſt in finiſhing its courſe round the ſun. The period of Saturn falls ſhort only ſixteen days of 29 years and a half. The period of Jupiter is twelve years wanting about 50 days. The period of Mars falls ſhort of two years by about 43 days. The revolution of the earth conſtitutes the year. Venus performs her period in about $224\frac{1}{2}$ days, and mercury in about 88 days.

5. The courſe of each planet lies throughout in one plane or flat ſurface, in which the ſun is placed; but they do not all move in the ſame plane, though the different planes, in which they move, croſs each other in very ſmall angles. They all croſs each other in lines, which paſs through the ſun; becauſe the ſun lies in the plane of each orbit. This inclination of the ſeveral orbits to each other is repreſented in fig. 90. The line, in which the plane of any orbit croſſes the plane of the earth's motion, is called the line of the nodes of that orbit.

6. Each planet moves round the ſun in the line, which we have mentioned above[a] under the name of ellipſis; which I ſhall here ſhew more particularly how to deſcribe. I have there ſaid how it is produced in the cone. I ſhall now ſhew how to form it upon a plane. Fix upon any plane two pins, as at A and B in fig. 91. To theſe tye a ſtring ACB of any length. Then apply a third pin D ſo to the ſtring, as to hold it ſtrained; and in that manner carrying this pin about, the point of it will deſcribe an ellipſis. If through the points A, B the ſtraight line EABF be drawn, to be terminated at the ellipſis in the points E and F, this is the longeſt line of any, that can be drawn within the figure, and is called the greater axis of the ellipſis. The line GH, drawn perpendicular to this axis EF, ſo as to paſs through the middle of it, is called the leſſer axis. The two points A and B are called focus's. Now each planet moves round the ſun in a line of this kind, ſo that the ſun is found in one focus. Suppoſe A to be the place of the ſun. Then E is the point, wherein the planet will be neareſt of all to the ſun, and at F it will be moſt remote. The point E is called the perihelion of the planet, and F the aphelion. In G and H the planet is ſaid to be in its middle or mean diſtance; becauſe the diſtance AG or AH is truly the middle between AE the leaſt, and AF the greateſt diſtance. In fig. 92. is repreſented how the greater axis of each orbit is ſituated in reſpect of the reſt. The proportion between the greateſt and leaſt diſtances of the planet from the ſun is very different in the different planets. In Saturn the proportion of the

greateſt

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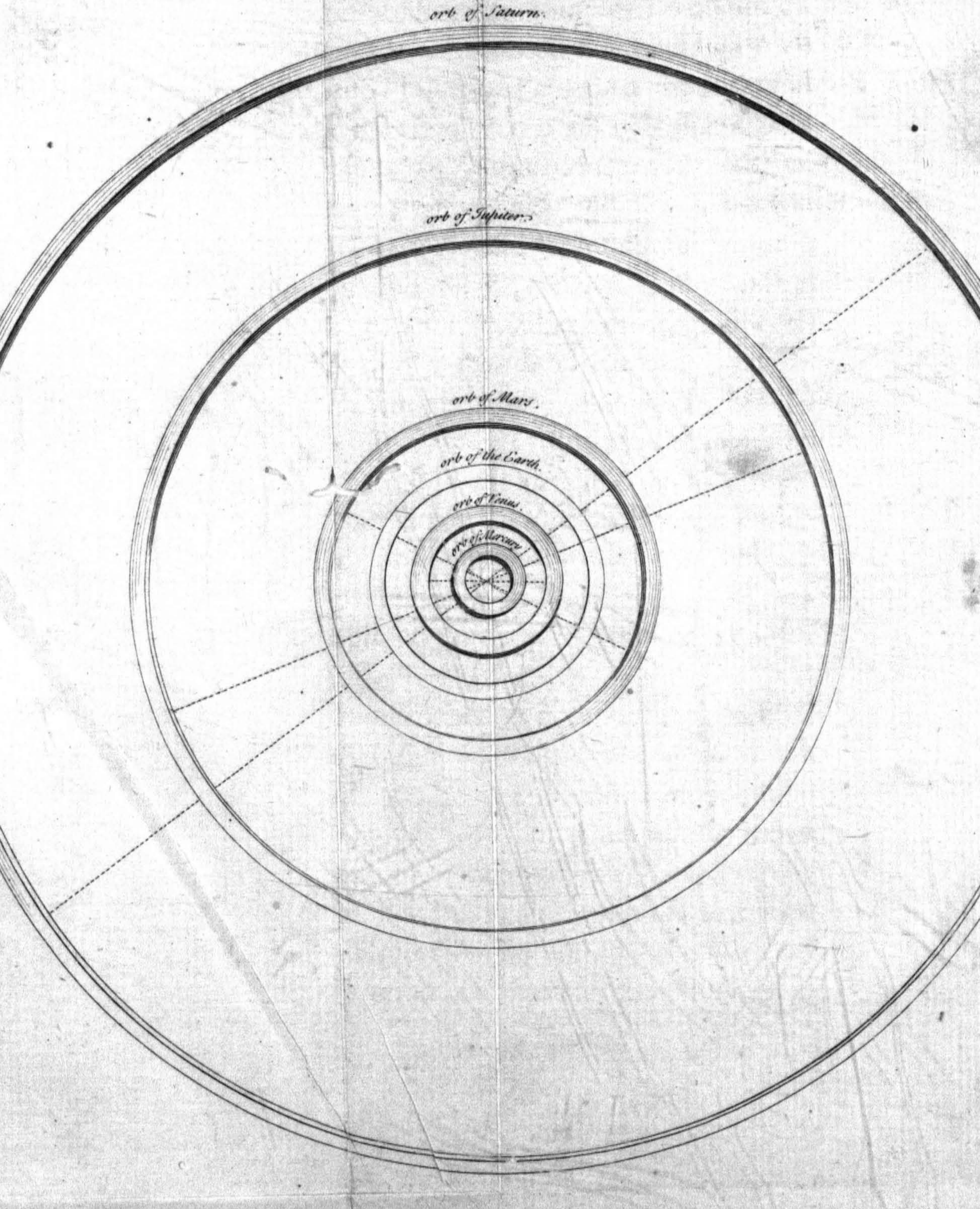
[a] Book I. ch. 2. § 82.

Fig. 90


orb of Saturn
orb of Jupiter
orb of Mars
orb of the Earth
orb of Venus
orb of Mercury

greateſt diſtance to the leaſt is ſomething leſs, than the proportion of 9 to 8; but much nearer to this, than to the proportion of 10 to 9. In Jupiter this proportion is a little greater, than that of 11 to 10. In Mars it exceeds the proportion of 6 to 5. In the earth it is about the proportion of 30 to 29. In Venus it is near to that of 70 to 69. And in Mercury it comes not a great deal ſhort of the proportion of 3 to 2.

7. EACH of theſe planets ſo moves through its ellipſis, that the line drawn from the ſun to the planet, by accompanying the planet in its motion, will deſcribe about the ſun equal ſpaces in equal times, after the manner ſpoke of in the chapter of centripetal forces[a]: There is alſo a certain relation between the greater axis's of theſe ellipſis's, and the times, in which the planets perform their revolutions through them. Which relation may be expreſſed thus. Let the period of one planet be denoted by the letter A, the greater axis of its orbit by D; let the period of another planet be denoted by B, and the greater axis of this planet's orbit by E. Then if C be taken to bear the ſame proportion to B, as B bears to A; likewiſe if F be taken to bear the ſame proportion to E, as E bears to D; and G taken to bear the ſame proportion likewiſe to F, as E bears to D; then A ſhall bear the ſame proportion to C, as D bears to G.

A	D
B	E
C	F
	G

8. THE ſecondary planets move round their reſpective primary, much in the ſame manner as the primary do round

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[a] Book I. Ch. 3. § 29.

the ſun. But the motions of theſe ſhall be more fully explained hereafter[a]. And there is, beſides the planets, another ſort of bodies, which in all probability move round the ſun; I mean the comets. The farther deſcription of which bodies I alſo leave to the place, where they are to be particularly treated on[b].

9. FAR without this ſyſtem the fixed ſtars are placed. Theſe are all ſo remote from us, that we ſeem almoſt incapable of contriving any means to eſtimate their diſtance. Their number is exceeding great. Beſides two or three thouſand, which we ſee with the naked eye, teleſcopes open to our view vaſt numbers; and the farther improved theſe inſtruments are, we ſtill diſcover more and more. Without doubt theſe are luminous globes, like our ſun, and ranged through the wide extent of ſpace; each of which, it is to be ſuppoſed, perform the ſame office, as our ſun, affording light and heat to certain planets moving about them. But theſe conjectures are not to be purſued in this place.

10. I SHALL therefore now proceed to the particular deſign of this chapter, and ſhew, that there is no ſenſible matter lodged in the ſpace where the planets move.

11. THAT they ſuffer no ſenſible reſiſtance from any ſuch matter, is evident from the agreement between the obſervations of aſtronomers in different ages, with regard to the time, in which the planets have been found to perform their

[a] Ch. 3. of this preſent book.

[b] Ch. 4.

periods. But it was the opinion of DES CARTES[a], that the planets might be kept in their courſes by the means of a fluid matter, which continually circulating round ſhould carry the planets along with it. There is one appearance that may ſeem to favour this opinion; which is, that the ſun turns round its own axis the ſame way, as the planets move. The earth alſo turns round its axis the ſame way, as the moon moves round the earth. And the planet Jupiter turns upon its axis the ſame way, as his ſatellites revolve round him. It might therefore be ſuppoſed, that if the whole planetary region were filled with a fluid matter, the ſun, by turning round on its own axis, might communicate motion firſt to that part of the fluid, which was contiguous, and by degrees propagate the like motion to the parts more remote. After the ſame manner the earth might communicate motion to this fluid, to a diſtance ſufficient to carry round the moon, and Jupiter communicate the like to the diſtance of its ſatellites. Sir ISAAC NEWTON has particularly examined what might be the reſult of ſuch a motion as this[b]; and he finds, that the velocities, with which the parts of this fluid will move in different diſtances from the center of the motion, will not agree with the motion obſerved in different planets: for inſtance, that the time of one intire circulation of the fluid, wherein Jupiter ſhould ſwim, would bear a greater proportion to the time of one intire circulation of the fluid, where the earth is; than the period of Jupiter bears to the period of the earth. But he alſo proves[c], that the planet cannot circulate in ſuch a fluid, ſo as to keep long in the ſame courſe, unleſs the planet and the contiguous fluid are of the ſame denſity, and the planet be carried along with the ſame degree of motion, as the fluid. There is alſo another remark made upon this motion by our author; which is, that ſome vivifying force will be continually neceſſary at the center of the motion [a]. The ſun in particular, by communicating motion to the ambient fluid, will loſe from it ſelf as much motion, as it imparts to the fluid; unleſs ſome acting principle reſide in the ſun to renew its motion continually. If the fluid be infinite, this gradual loſs of motion would continue till the whole ſhould ſtop [b]; and if the fluid were limited, this loſs of motion would continue, till there would remain no ſwifter a revolution in the ſun, than in the utmoſt part of the fluid; ſo that the whole would turn together about the axis of the ſun, like one ſolid globe [c].

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[a] In Princ. philoſ. part. 3.

[b] Philoſ. princ. mathem. Lib. II. prop. 2. & ſchol.

[c] Ibid. prop. 53.

12. IT is farther to be obſerved, that as the planets do not move in perfect circles round the ſun; there is a greater diſtance between their orbits in ſome places, than in others. For inſtance, the diſtance between the orbit of Mars and Venus is near half as great again in one part of their orbits, as in the oppoſite place. Now here the fluid, in which the earth ſhould ſwim, muſt move with a leſs rapid motion, where there is this greater interval between the contiguous orbits; but on the contrary, where the ſpace is ſtraiteſt, the earth moves more ſlowly, than where it is wideſt [d].

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[a] Philoſ. princ. prop. 52. coroll. 4.

[b] Ibid.

[c] Coroll. 11.

[d] See ibid. ſchol. poſt prop. 53.

13. FARTHER, if this our globe of earth ſwam in a fluid of equal denſity with the earth it ſelf, that is, in a fluid more denſe than water; all bodies put in motion here upon the earth's ſurface muſt ſuffer a great reſiſtance from it; where as, by Sir ISAAC NEWTON's experiments mentioned in the preceding chapter, bodies, that fell perpendicularly down through the air, felt about $\frac{1}{860}$ part only of the reſiſtance, which bodies ſuffered that fell in like manner through water.

14. Sir ISAAC NEWTON applies theſe experiments yet farther, and examines by them the general queſtion concerning the abſolute plenitude of ſpace. According to the Ariſtotelians, all ſpace was full without any the leaſt vacuities whatever. DES CARTES embraced the ſame opinion, and therefore ſuppoſed a ſubtile fluid matter, which ſhould pervade all bodies, and adequately fill up their pores. The Atomical philoſophers, who ſuppoſe all bodies both fluid and ſolid to be compoſed of very minute but ſolid atoms, aſſert that no fluid, how ſubtile ſoever the particles or atoms whereof it is compoſed ſhould be, can ever cauſe an abſolute plenitude; becauſe it is impoſſible that any body can paſs through the fluid without putting the particles of it into ſuch a motion, as to ſeparate them, at leaſt in part, from one another, and ſo perpetually to cauſe ſmall vacuities; by which theſe Atomiſts endeavour to prove, that a vacuum, or ſome ſpace empty of all matter, is abſolutely neceſſary to be in nature. Sir ISAAC NEWTON objects againſt the filling of ſpace with ſuch a ſubtile fluid, that all bodies in motion muſt be unmeaſurably re-

ſiſted by a fluid ſo denſe, as abſolutely to fill up all the ſpace, through which it is ſpread. And leſt it ſhould be thought, that this objection might be evaded by aſcribing to this fluid ſuch very minute and ſmooth parts, as might remove all adheſion or friction between them, whereby all reſiſtance would be loſt, which this fluid might otherwiſe give to bodies moving in it; Sir ISAAC NEWTON proves, in the manner above related, that fluids reſiſt from the power of inactivity of their particles; and that water and the air reſiſt almoſt entirely on this account: ſo that in this ſubtile fluid, however minute and lubricated the particles, which compoſe it, might be; yet if the whole fluid was as denſe as water, it would reſiſt very near as much as water does; and whereas ſuch a fluid, whoſe parts are abſolutely cloſe together without any intervening ſpaces, muſt be a great deal more denſe than water, it muſt reſiſt more than water in proportion to its greater denſity; unleſs we will ſuppoſe the matter, of which this fluid is compoſed, not to be endued with the ſame degree of inactivity as other matter. But if you deprive any ſubſtance of the property ſo univerſally belonging to all other matter, without impropriety of ſpeech it can ſcarce be called by this name.

15. Sir ISAAC NEWTON made alſo an experiment to try in particular, whether the internal parts of bodies ſuffered any reſiſtance. And the reſult did indeed appear to favour ſome ſmall degree of reſiſtance; but ſo very little, as to leave it doubtful, whether the effect did not ariſe from ſome other latent cauſe[a].

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[a] Princ. philoſ. pag. 316, 317.

CHAP.

CHAP. II.

Concerning the cauſe, which keeps in motion the primary planets.

SINCE the planets move in a void ſpace and are free from reſiſtance; they, like all other bodies, when once in motion, would move on in a ſtraight line without end, if left to themſelves. And it is now to be explained what kind of action upon them carries them round the ſun. Here I ſhall treat of the primary planets only, and diſcourſe of the ſecondary apart in the next chapter. It has been juſt now declared, that theſe primary planets move ſo about the ſun, that a line extended from the ſun to the planet, will, by accompanying the planet in its motion, paſs over equal ſpaces in equal portions of time [a]. And this one property in the motion of the planets proves, that they are continually acted on by a power directed perpetually to the ſun as a center. This therefore is one property of the cauſe, which keeps the planets in their courſes, that it is a centripetal power, whoſe center is the ſun.

2. AGAIN, in the chapter upon centripetal forces [b] it was obſerv'd, that if the ſtrength of the centripetal power was ſuitably accommodated every where to the motion of any body round a center, the body might be carried in

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[a] Ch. 1. § 7.

[b] Book I. Ch. 3.

any bent line whatever, whoſe concavity ſhould be every where turned towards the center of the force. It was farther remarked, that the ſtrength of the centripetal force, in each place, was to be collected from the nature of the line, wherein the body moved[a]. Now ſince each planet moves in an ellipſis, and the ſun is placed in one focus; Sir ISAAC NEWTON deduces from hence, that the ſtrength of this power is reciprocally in the duplicate proportion of the diſtance from the ſun. This is deduced from the properties, which the geometers have diſcovered in the ellipſis. The proceſs of the reaſoning is not proper to be enlarged upon here; but I ſhall endeavour to explain what is meant by the reciprocal duplicate proportion. Each of the terms reciprocal proportion, and duplicate proportion, has been already defined[b]. Their ſenſe when thus united is as follows. Suppoſe the planet moved in the orbit ABC (in fig. 93.) about the ſun in S. Then, when it is ſaid, that the centripetal power, which acts on the planet in A, bears to the power acting on it in B a proportion, which is the reciprocal of the duplicate proportion of the diſtance SA to the diſtance SB; it is meant that the power in A bears to the power in B the duplicate of the proportion of the diſtance SB to the diſtance SA. The reciprocal duplicate proportion may be explained alſo by numbers as follows. Suppoſe ſeveral diſtances to bear to each other proportions expreſſed by the numbers 1, 2, 3, 4, 5; that is, let the ſecond diſtance be double the firſt, the third be three times, the fourth four times, and the fifth five times as great as the firſt. Multiply each of theſe numbers by it ſelf, and 1 multiplied by 1 produces ſtill 1, 2 multiplied by 2 produces 4, 3 by 3 makes 9, 4 by 4 makes 16, and 5 by 5 gives 25. This being done, the fractions $\frac{1}{4}$, $\frac{1}{9}$, $\frac{1}{16}$, $\frac{1}{25}$, will reſpectively expreſs the proportion, which the centripetal power in each of the following diſtances bears to the power at the firſt diſtance: for in the ſecond diſtance, which is double the firſt, the centripetal power will be one fourth part only of the power at the firſt diſtance; at the third diſtance the power will be one ninth part only of the firſt power; at the fourth diſtance, the power will be but one ſixteenth part of the firſt; and at the fifth diſtance, one twenty fifth part of the firſt power.

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[a] Book I. Ch. 3. § 29.

[b] Ibid. Ch. 2. § 30, 17.

3. THUS is found the proportion, in which this centripetal power decreaſes, as the diſtance from the ſun increaſes, within the compaſs of one planet's motion. How it comes to paſs, that the planet can be carried about the ſun by this centripetal power in a continual round, ſometimes riſing from the ſun, then deſcending again as low, and from thence be carried up again as far remote as before, alternately riſing and falling without end; appears from what has been written above concerning centripetal forces: for the orbits of the planets reſemble in ſhape the curve line propoſed in § 17 of the chapter on theſe forces [a].

4. BUT farther, in order to know whether this centripetal force extends in the ſame proportion throughout, and conſequently whether all the planets are influenced by the very ſame

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[a] Book I. Ch. 3.

power, our author proceeds thus. He inquires what relation there ought to be between the periods of the different planets, provided they were acted upon by the ſame power decreaſing throughout in the forementioned proportion; and he finds, that the period of each in this caſe would have that very relation to the greater axis of its orbit, as I have declared above[a] to be found in the planets by the obſervations of aſtronomers. And this puts it beyond queſtion, that the different planets are preſſed towards the ſun, in the ſame proportion to their diſtances, as one planet is in its ſeveral diſtances. And thence in the laſt place it is juſtly concluded, that there is ſuch a power acting towards the ſun in the foreſaid proportion at all diſtances from it.

5. THIS power, when referred to the planets, our author calls centripetal, when to the ſun attractive; he gives it likewiſe the name of gravity, becauſe he finds it to be of the ſame nature with that power of gravity, which is obſerved in our earth, as will appear hereafter[b]. By all theſe names he deſigns only to ſignify a power endued with the properties before mentioned; but by no means would he have it underſtood, as if theſe names referred any way to the cauſe of it. In particular in one place where he uſes the name of attraction, he cautions us expreſsly againſt implying any thing but a power directing a body to a center without any reference to the cauſe of it, whether reſiding in that center, or ariſing from any external impulſe[c].

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[a] Ch. 1. § 7.

[b] Chap. 5. § 8.

[c] Princ. pag. 60.

6. BUT now, in theſe demonſtrations ſome very minute inequalities in the motion of the planets are neglected; which is done with a great deal of judgment; for whatever be their cauſe, the effects are very inconſiderable, they being ſo exceeding ſmall, that ſome aſtronomers have thought fit wholly to paſs them by [a]. However the excellency of this philoſophy, when in the hands of ſo great a geometer as our author, is ſuch, that it is able to trace the leaſt variations of things up to their cauſes. The only inequalities, which have been obſerved common to all the planets, are the motion of the aphelion and the nodes The tranſverſe axis of each orbit does not always remain fixed, but moves about the ſun with a very ſlow progreſſive motion: nor do the planets keep conſtantly the ſame plane, but change them, and the lines in which thoſe planes interſect each other by inſenſible degrees. The firſt of theſe inequalities, which is the motion of the aphelion, may be accounted for, by ſuppoſing the gravitation of the planets towards the ſun to differ a little from the forementioned reciprocal duplicate proportion of the diſtances; but the ſecond, which is the motion of the nodes, cannot be accounted for by any power directed towards the ſun; for no ſuch can give the planet any lateral impulſe to divert it from the plane of its motion into any new plane, but of neceſſity muſt be derived from ſome other center. Where that power is lodged, remains to be diſcovered. Now it is proved, as ſhall be explained in the following chapter, that the three primary planets Saturn, Jupiter, and the earth, which have ſatellites revolving about them, are endued with a power of

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[a] Street, in Aſtron. Carolin.

cauſing bodies, in particular thoſe ſatellites, to gravitate towards them with a force, which is reciprocally in the duplicate proportion of their diſtances; and the planets are in all reſpects, in which they come under our examination, ſo ſimilar and alike, that there is no reaſon to queſtion, but they have all the ſame property. Though it be ſufficient for the preſent purpoſe to have it proved of Jupiter and Saturn only; for theſe planets contain much greater quantities of matter than the reſt, and proportionally exceed the others in power [a]. But the influence of theſe two planets being allowed, it is evident how the planets come to ſhift continually their planes: for each of the planets moving in a different plane, the action of Jupiter and Saturn upon the reſt will be oblique to the planes of their motion; and therefore will gradually draw them into new ones. The ſame action of theſe two planets upon the reſt will cauſe likewiſe a progreſſive motion of the aphelion; ſo that there will be no neceſſity of having recourſe to the other cauſe for this motion, which was before hinted at [b]; viz, the gravitation of the planets towards the ſun differing from the exact reciprocal duplicate proportion of the diſtances. And in the laſt place, the action of Jupiter and Saturn upon each other will produce in their motions the ſame inequalities, as their joint action produces in the reſt. All this is effected in the ſame manner, as the ſun produces the ſame kind of inequalities and many others in the motion of the moon and the other ſecondary planets; and therefore will be beſt apprehended by what ſhall be ſaid in the next chapter.

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[a] See Chap. 5. § 9, &c.

[b] In the foregoing page.

Thoſe other irregularities in the motion of the ſecondary planets have place likewiſe here; but are too minute to be obſervable: becauſe they are produced and rectified alternately, for the moſt part in the time of a ſingle revolution; whereas the motion of the aphelion and nodes, which continually increaſe, become ſenſible in a long ſeries of years. Yet ſome of theſe other inequalities are diſcernible in Jupiter and Saturn, in Saturn chiefly; for when Jupiter, who moves faſter than Saturn, approaches near to a conjunction with him, his action upon Saturn will a little retard the motion of that planet, and by the reciprocal action of Saturn he will himſelf be accelerated. After conjunction, Jupiter will again accelerate Saturn, and be likewiſe retarded in the ſame degree, as before the firſt was retarded and the latter accelerated. Whatever inequalities beſides are produced in the motion of Saturn by the action of Jupiter upon that planet, will be ſufficiently rectified, by placing the focus of Saturn's ellipſis, which ſhould otherwiſe be in the ſun, in the common center of gravity of the ſun and Jupiter. And all the inequalities in the motion of Jupiter, cauſed by Saturn's action upon him, are much leſs conſiderable than the irregularities of Saturn's motion[a].

7. THIS one principle therefore of the planets having a power, as well as the ſun, to cauſe bodies to gravitate towards them, which is proved by the motion of the ſecondary planets to obtain in fact, explains all the irregularities relating to the planets ever obſerved by aſtronomers.

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[a] See Newton. Princ. Lib. III. prop. 13.

8. Sir ISAAC NEWTON after this proceeds to make an improvement in aſtronomy by applying this theory to the farther correction of their motions. For as we have here obſerved the planets to poſſeſs a principle of gravitation, as well as the ſun; ſo it will be explained at large hereafter, that the third law of motion, which makes action and reaction equal, is to be applied in this caſe [a]; and that the ſun does not only attract each planet, but is it ſelf alſo attracted by them; the force, wherewith the planet is acted on, bearing to the force, wherewith the ſun it ſelf is acted on at the ſame time, the proportion, which the quantity of matter in the ſun bears to the quantity of matter in the planet. From the action between the ſun and planet being thus mutual Sir ISAAC NEWTON proves that the ſun and planet will deſcribe about their common center of gravity ſimilar ellipſis's; and then that the tranſverſe axis of the ellipſis deſcribed thus about the moveable ſun, will bear to the tranſverſe axis of the ellipſis, which would be deſcribed about the ſun at reſt in the ſame time, the ſame proportion as the quantity of ſolid matter in the ſun and planet together bears to the firſt of two mean proportionals between this quantity and the quantity of matter in the ſun only [b].

9. ABOVE, where I ſhewed how to find a cube, that ſhould bear any proportion to another cube [c], the lines FT and TS are two mean proportionals between EF and FG; and counting from EF, FT is called the firſt, and FS the ſecond of thoſe means. In numbers theſe mean proportionals

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[a] Chap. 5. § 10.

[b] Princ. Lib. I. prop. 60.

[c] Book I. Chap. 2. § 80.

are thus found. Suppoſe A and B two numbers, and it be required to find C the firſt, and D the ſecond of the two mean proportionals between them. Firſt multiply A by it ſelf, and the product multiply by B; then C will be the number which in arithmetic is called the cubic root of this laſt product; that is, the number C being multiplied by it ſelf, and the product again multiplied by the ſame number C, will produce the product above mentioned. In like manner D is the cubic root of the product of B multiplied by it ſelf, and the produce of that multiplication multiplied again by A.

A C
B D

10. IT will be asked, perhaps, how this correction can be admitted, when the cauſe of the motions of the planets was before found by ſuppoſing the ſun the center of the power, which acted upon them: for according to the preſent correction this power appears rather to be directed to their common center of gravity. But whereas the ſun was at firſt concluded to be the center, to which the power acting on the planets was directed, becauſe the ſpaces deſcribed round the ſun in equal times were found to be equal; ſo Sir ISAAC NEWTON proves, that if the ſun and planet move round their common center of gravity, yet to an eye placed in the planet, the ſpaces, which will appear to be deſcribed about the ſun, will have the ſame relation to the times of their deſcription, as the real ſpaces would have, if the ſun were at reſt[a]. I farther aſſerted, that, ſuppoſing the planets to move round the ſun at reſt,

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[a] Princ. philoſ. Lib. I. prop. 58. coroll. 3.

and to be attracted by a power, which every where ſhould act with degrees of ſtrength reciprocally in the duplicate proportion of the diſtances; then the periods of the planets muſt obſerve the ſame relation to their diſtances, as aſtronomers find them to do. But here it muſt not be ſuppoſed, that the obſervations of aſtronomers abſolutely agree without any the leaſt difference; and the preſent correction will not cauſe a deviation from any one aſtronomer's obſervations, ſo much as they differ from one another. For in Jupiter, where this correction is greateſt, it hardly amounts to the 3000th part of the whole axis.

11. Upon this head I think it not improper to mention a reflection made by our excellent author upon theſe ſmall inequalities in the planets motions; which contains under it a very ſtrong philoſophical argument againſt the eternity of the world. It is this, that theſe inequalities muſt continually increaſe by ſlow degrees, till they render at length the preſent frame of nature unfit for the purpoſes, it now ſerves [a]. And a more convincing proof cannot be deſired againſt the preſent conſtitution's having exiſted from eternity than this, that a certain period of years will bring it to an end. I am aware this thought of our author has been repreſented even as impious, and as no leſs than caſting a reflection upon the wiſdom of the author of nature, for framing a periſhable work. But I think ſo bold an aſſertion ought to have been made with ſingular caution. For if this remark upon the increaſing irregularities of the heavenly motions

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[a] Newt. Optics, pag. 378.

be true in fact, as it really is, the imputation muſt return upon the aſſerter, that this does detract from the divine wiſdom. Certainly we cannot pretend to know all the omniſcient Creator's purpoſes in making this world, and therefore cannot undertake to determine how long he deſigned it ſhould laſt. And it is ſufficient, if it endure the time intended by the author. The body of every animal ſhews the unlimited wiſdom of its author no leſs, nay in many reſpects more, than the larger frame of nature; and yet we ſee, they are all deſigned to laſt but a ſmall ſpace of time.

12. There need nothing more be ſaid of the primary planets; the motions of the ſecondary ſhall be next conſidered.

CHAP. III.

Of the motion of the MOON and the other SECONDARY PLANETS.

THE excellency of this philoſophy ſufficiently appears from its extending in the manner, which has been related, to the minuteſt circumſtances of the primary planets motions; which nevertheleſs bears no proportion to the vaſt ſucceſs of it in the motions of the ſecondary; for it not only accounts for all the irregularities, by which their motions were known to be diſturbed, but has diſcovered others ſo complicated, that aſtronomers were never able to diſtinguiſh them, and reduce them under proper heads; but theſe were only to be found out from their cauſes, which this philoſophy has brought to light, and has ſhewn the dependence of theſe inequalities upon ſuch cauſes in ſo perfect a manner, that we not only learn from thence in general, what thoſe inequalities are, but are able to compute the degree of them. Of this Sir Iſ. Newton has given ſeveral ſpecimens, and has moreover found means to reduce the moon's motion ſo completely to rule, that he has framed a theory, from which the place of that planet may at all times be computed, very nearly or altogether as exactly, as the places of the primary planets themſelves, which is much beyond what the greateſt aſtronomers could ever effect.

2. The firſt thing demonſtrated of theſe ſecondary planets is, that they are drawn towards their reſpective primary in the ſame manner as the primary planets are attracted by the ſun. That each ſecondary planet is kept in its orbit by a power pointed towards the center of the primary planet, about which the ſecondary revolves; and that the power, by which the ſecondaries of the ſame primary are influenced, bears the ſame relation to the diſtance from the primary, as the power, by which the primary planets are guided, does in regard to the diſtance from the ſun[a]. This is proved in the ſatellites of Jupiter and Saturn, becauſe they move in circles, as far as we can obſerve, about their reſpective primary with an equable courſe, the reſpective primary being the center of each orbit: and by comparing the times, in which the different ſatellites of the ſame primary perform their periods, they are

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[a] Newton. Princ. Lib. III. prop. 1.

found to obſerve the ſame relation to the diſtances from their primary, as the primary planets obſerve in reſpect of their mean diſtances from the ſun[a]. Here theſe bodies moving in circles with an equable motion, each ſatellite paſſes over equal parts of its orbit in equal portions of time; conſequently the line drawn from the center of the orbit, that is, from the primary planet, to the ſatellite, will paſs over equal ſpaces along with the ſatellite in equal portions of time; which proves the power, by which each ſatellite is held in its orbit, to be pointed towards the primary as a center[b]. It is alſo manifeſt that the centripetal power, which carries a body in a circle concentrical with the power, acts upon the body at all times with the ſame ſtrength. But Sir ISAAC NEWTON demonſtrates that, when bodies are carried in different circles by centripetal powers directed to the centers of thoſe circles, then the degrees of ſtrength of thoſe powers are to be compared by conſidering the relation between the times, in which the bodies perform their periods through thoſe circles[c]; and in particular he ſhews, that if the periodical times bear that relation, which I have juſt now aſſerted the ſatellites of the ſame primary to obſerve; then the centripetal powers are reciprocally in the duplicate proportion of the ſemidiameters of the circles, or in that proportion to the diſtances of the bodies from the centers[d]. Hence it follows that in the planets Jupiter and Saturn, the centripetal power in each decreaſes with the increaſe of diſtance, in the ſame proportion as the centripetal power appertaining to the ſun decreaſes with the increaſe of diſtance. I do not here mean that this proportion of the centripetal powers holds between the power of Jupiter at any diſtance compared with the power of Saturn at any other diſtance; but only in the change of ſtrength of the power belonging to the ſame planet at different diſtances from him. Moreover what is here diſcovered of the planets Jupiter and Saturn by means of the different ſatellites, which revolve round each of them, appears in the earth by the moon alone; becauſe ſhe is found to move round the earth in an ellipſis after the ſame manner as the primary planets do about the ſun; excepting only ſome ſmall irregularities in her motion, the cauſe of which will be particularly explained in what follows, whereby it will appear, that they are no objection againſt the earth's acting on the moon in the ſame manner as the ſun acts on the primary planets; that is, as the other primary planets Jupiter and Saturn act upon their ſatellites. Certainly ſince theſe irregularities can be otherwiſe accounted for, we ought not to depart from that rule of induction ſo neceſſary in philoſophy, that to like bodies like properties are to be attributed, where no reaſon to the contrary appears. We cannot therefore but aſcribe to the earth the ſame kind of action upon the moon, as the other primary planets Jupiter and Saturn have upon their ſatellites; which is known to be very exactly in the proportion aſſigned by the method of comparing the periodical times and diſtances of all the ſatellites, which move about the ſame planet; this abundantly compenſating our not being near enough to obſerve the exact figure of their orbits. For if the little deviation of the moon's orbit from a true permanent ellipſis aroſe from the action of the earth upon the moon not being in the exact reciprocal duplicate proportion of the diſtance, were another moon to revolve about the earth, the proportion between the periodical times of this new moon, and the preſent, would diſcover the deviation from the mentioned proportion much more manifeſtly.

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[a] Newton. Princ. Lib. III. pag. 390, 391. compared with pag. 393.

[b] Book I. Ch. 3. § 29.

[c] Princ. philoſ. Lib. I. prop. 4.

[d] Ibid. coroll.

3. BY the number of ſatellites, which move round Jupiter and Saturn, the power of each of theſe planets is meaſured in a great diverſity of diſtance; for the diſtance of the outermoſt ſatellite in each of theſe planets exceeds ſeveral times the diſtance of the innermoſt. In Jupiter the aſtronomers have uſually placed the innermoſt ſatellite at a diſtance from the center of that planet equal to about $5\frac{2}{3}$ of the ſemidiameters of Jupiter's body, and this ſatellite performs its revolution in about 1 day $18\frac{1}{2}$ hours. The next ſatellite, which revolves round Jupiter in about 3 days $13\frac{1}{5}$ hours, they place at the diſtance from Jupiter of about 9 of that planet's ſemidiameters. To the third ſatellite, which performs its period nearly in 7 days $3\frac{3}{4}$ hours, they aſſign the diſtance of about $14\frac{2}{5}$ ſemidiameters. But the outermoſt ſatellite they remove to $25\frac{1}{3}$ ſemidiameters, and this ſatellite makes its period in about 16 days $16\frac{1}{2}$ hours[a]. In Saturn there is ſtill a greater diverſity in the diſtance of the ſeveral ſatellites. By the obſervations of the late CASSINI, a celebrated aſtronomer in France, who firſt diſcovered all theſe ſatellites, except one known before, the innermoſt is diſtant about $4\frac{1}{2}$ of Saturn's ſemidiameters from his center, and re-

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[a] Newt. Princ. philoſ. Lib. III. pag. 390.

volves round in about 1 day $21\frac{1}{3}$ hours. The next ſatellite is diſtant about $5\frac{3}{4}$ ſemidiameters, and makes its period in about 2 days $17\frac{2}{3}$ hours. The third is removed to the diſtance of about 8 ſemidiameters, and performs its revolution in near 4 days $12\frac{1}{2}$ hours. The fourth ſatellite diſcovered firſt by the great Huygens, is near $18\frac{2}{3}$ ſemidiameters, and moves round Saturn in about 15 days $22\frac{2}{3}$ hours. The outermoſt is diſtant 56 ſemidiameters, and makes its revolution in about 79 days $7\frac{4}{5}$ hours[a]. Beſides theſe ſatellites, there belongs to the planet Saturn another body of a very ſingular kind. This is a ſhining, broad, and flat ring, which encompaſſes the planet round. The diameter of the outermoſt verge of this ring is more than double the diameter of Saturn. Huygens, who firſt deſcribed this ring, makes the whole diameter thereof to bear to the diameter of Saturn the proportion of 9 to 4. The late reverend Mr. Pound makes the proportion ſomething greater, viz. that of 7 to 3. The diſtances of the ſatellites of this planet Saturn are compared by Cassini to the diameter of the ring. His numbers I have reduced to thoſe above, according to Mr. Pound's proportion between the diameters of Saturn and of his ring. As this ring appears to adhere no where to Saturn, ſo the diſtance of Saturn from the inner edge of the ring ſeems rather greater than the breadth of the ring. The diſtances, which have here been given, of the ſeveral ſatellites, both for Jupiter and Saturn, may be more depended on in relation to the proportion, which thoſe belonging to the ſame primary planet bear one to another, than in reſpect to the very numbers, that have been here ſet down, by reaſon of the difficulty there is in meaſuring to the greateſt exactneſs the diameters of the primary planets; as will be explained hereafter, when we come to treat of teleſcopes [a]. By the obſervations of the forementioned Mr. POUND, in Jupiter the diſtance of the innermoſt ſatellite ſhould rather be about 6 ſemidiameters, of the ſecond $9\frac{1}{2}$, of the third 15, and of the outermoſt $26\frac{2}{3}$ [b]; and in Saturn the diſtance of the innermoſt ſatellite 4 ſemidiameters, of the next $6\frac{1}{4}$, of the third $8\frac{3}{4}$, of the fourth $20\frac{1}{3}$, and of the fifth 59 [c]. However the proportion between the diſtances of the ſatellites in the ſame primary is the only thing neceſſary to the point we are here upon.

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[a] Newt. Princ. philoſ. Lib. III. pag. 391, 392.

4. BUT moreover the force, wherewith the earth acts in different diſtances, is confirmed from the following conſideration, yet more expreſly than by the preceding analogical reaſoning. It will appear, that if the power of the earth, by which it retains the moon in her orbit, be ſuppoſed to act at all diſtances between the earth and moon, according to the forementioned rule; this power will be ſufficient to produce upon bodies, near the ſurface of the earth, all the effects aſcribed to the principle of gravity. This is diſcovered by the following method. Let A (in fig. 94.) repreſent the earth, B the moon, BCD the moon's orbit, which differs little from a circle, of which A is the center. If the moon in B were left to it ſelf to move with the velocity, it has in the point B, it

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[a] Book III. Ch. 4.

[b] Newt. Princ. philoſ. Lib. III. pag. 391.

[c] Ibid. pag. 392.

would leave the orbit, and proceed right forward in the line BE, which touches the orbit in B. Suppoſe the moon would upon this condition move from B to E in the ſpace of one minute of time. By the action of the earth upon the moon, whereby it is retained in its orbit, the moon will really be found at the end of this minute in the point F, from whence a ſtraight line drawn to A ſhall make the ſpace BFA in the circle equal to the triangular ſpace BEA; ſo that the moon in the time wherein it would have moved from B to E, if left to it ſelf, has been impelled towards the earth from E to F. And when the time of the moon's paſſing from B to F is ſmall, as here it is only one minute, the diſtance between E and F ſcarce differs from the ſpace, through which the moon would deſcend in the ſame time, if it were to fall directly down from B toward A without any other motion. AB the diſtance of the earth and moon is about 60 of the earth's ſemidiameters, and the moon completes her revolution round the earth in about 27 days 7 hours and 43 minutes: therefore the ſpace EF will here be found by computation to be about $16\frac{1}{8}$ feet. Conſequently, if the power, by which the moon is retained in its orbit, be near the ſurface of the earth greater, than at the diſtance of the moon in the duplicate proportion of that diſtance; the number of feet, a body would deſcend near the ſurface of the earth by the action of this power upon it in one minute of time, would be equal to $16\frac{1}{8}$ multiplied twice into the number 60, that is, equal to 58050. But how faſt bodies fall near the ſurface of the earth may be known by the pendulum[a]; and by the exacteſt experiments they are found to deſcend the ſpace of $16\frac{1}{8}$ feet in a ſecond of time; and the ſpaces deſcribed by falling bodies being in the duplicate proportion of the times of their fall [a], the number of feet, a body would deſcribe in its fall near the ſurface of the earth in one minute of time, will be equal to $16\frac{1}{8}$ twice multiplied by 60, the ſame as would be cauſed by the power which acts upon the moon.

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[a] See Book I. Ch. 2. § 60, 64.

5. In this computation the earth is ſuppoſed to be at reſt, whereas it would have been more exact to have ſuppoſed it to move, as well as the moon, about their common center of gravity; as will eaſily be underſtood, by what has been ſaid in the preceding chapter, where it was ſhewn, that the ſun is ſubjected to the like motion about the common center of gravity of it ſelf and the planets. The action of the ſun upon the moon, which is to be explain'd in what follows, is likewiſe here neglected: and Sir Isaac Newton ſhews, if you take in both theſe conſiderations, the preſent computation will beſt agree to a ſomewhat greater diſtance of the moon and earth, viz. to $60\frac{1}{2}$ ſemidiameters of the earth, which diſtance is more conformable to aſtronomical obſervations.

6. These computations afford an additional proof, that the action of the earth obſerves the ſame proportion to the diſtance, which is here contended for. Before I ſaid, it was reaſonable to conclude ſo by induction from the pla-

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[a] Book I. Ch. 2. § 17.

nets Jupiter and Saturn; becauſe they act in that manner. But now the ſame thing will be evident by drawing no other conſequence from what is ſeen in thoſe planets, than that the power, by which the primary planets act on their ſecondary, is extended from the primary through the whole interval between, ſo that it would act in every part of the intermediate ſpace. In Jupiter and Saturn this power is ſo far from being confined to a ſmall extent of diſtance, that it not only reaches to ſeveral ſatellites at very different diſtances, but alſo from one planet to the other, nay even through the whole planetary ſyſtem [a]. Conſequently there is no appearance of reaſon, why this power ſhould not act at all diſtances, even at the very ſurfaces of theſe planets as well as farther off. But from hence it follows, that the power, which retains the moon in her orbit, is the ſame, as cauſes bodies near the ſurface of the earth to gravitate. For ſince the power, by which the earth acts on the moon, will cauſe bodies near the ſurface of the earth to deſcend with all the velocity they are found to do, it is certain no other power can act upon them beſides; becauſe if it did, they muſt of neceſſity deſcend ſwifter. Now from all this it is at length very evident, that the power in the earth, which we call gravity, extends up to the moon, and decreaſes in the duplicate proportion of the increaſe of the diſtance from the earth.

7. This finiſhes the diſcoveries made in the action of the primary planets upon their ſecondary. The next thing

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[a] See Ch. II. § 6.

to be ſhewn is, that the ſun acts upon them likewiſe: for this purpoſe it is to be obſerved, that if to the motion of the ſatellite, whereby it would be carried round its primary at reſt, be ſuperadded the ſame motion both in regard to velocity and direction, as the primary it ſelf has, it will deſcribe about the primary the ſame orbit, with as great regularity, as if the primary was indeed at reſt. The cauſe of this is that law of motion, which makes a body near the ſurface of the earth, when let fall, to deſcend perpendicularly, though the earth be in ſo ſwift a motion, that if the falling body did not partake of it, its deſcent would be remarkably oblique; and that a body projected deſcribes in the moſt regular manner the ſame parabola, whether projected in the direction, in which the earth moves, or in the oppoſite direction, if the projecting force be the ſame [a]. From this we learn, that if the ſatellite moved about its primary with perfect regularity, beſides its motion about the primary, it would participate of all the motion of its primary; have the ſame progreſſive velocity, with which the primary is carried about the ſun; and be impelled with the ſame velocity as the primary towards the ſun, in a direction parallel to that impulſe of its primary. And on the contrary, the want of either of theſe, in particular of the impulſe towards the ſun, will occaſion great inequalities in the motion of the ſecondary planet. The inequalities, which would ariſe from the abſence of this impulſe towards the ſun are ſo great, that by the regularity, which appears in the motion of the ſecondary planets, it is proved, that the ſun communicates the ſame velocity to them by its action, as it gives to their primary at the ſame diſtance. For Sir ISAAC NEWTON informs us, that upon examination he found, that if any of the ſatellites of Jupiter were attracted by the ſun more or leſs, than Jupiter himſelf at the ſame diſtance, the orbit of that ſatellite, inſtead of being concentrical to Jupiter, muſt have its center at a greater or leſs diſtance, than the center of Jupiter from the ſun, nearly in the ſubduplicate proportion of the difference between the ſun's action upon the ſatellite, and upon Jupiter; and therefore if any ſatellite were attracted by the ſun but $\frac{1}{1000}$ part more or leſs, than Jupiter is at the ſame diſtance, the center of the orbit of that ſatellite would be diſtant from the center of Jupiter no leſs than a fifth part of the diſtance of the outermoſt ſatellite from Jupiter[a]; which is almoſt the whole diſtance of the innermoſt ſatellite. By the like argument the ſatellites of Saturn gravitate towards the ſun, as much as Saturn it ſelf at the ſame diſtance; and the moon as much as the earth.

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[a] The ſecond of the laws of motion laid down in Book I. Ch. 1.

8. THUS is proved, that the ſun acts upon the ſecondary planets, as much as upon the primary at the ſame diſtance: but it was found in the laſt chapter, that the action of the ſun upon bodies is reciprocally in the duplicate proportion of the diſtance; therefore the ſecondary

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[a] Newton. Princ. philoſ. Lib. III. prop. 6. pag. 401.

planets being ſometimes nearer to the ſun than the primary, and ſometimes more remote, they are not always acted upon in the ſame degree with their primary, but when nearer to the ſun, are attracted more, and when farther diſtant, are attracted leſs. Hence ariſe various inequalities in the motion of the ſecondary planets [a].

9. SOME of theſe inequalities would take place, though the moon, if undiſturbed by the ſun, would have moved in a circle concentrical to the earth, and in the plane of the earth's motion; others depend on the elliptical figure, and the oblique ſituation of the moon's orbit. One of the firſt kind is, that the moon is cauſed ſo to move, as not to deſcribe equal ſpaces in equal times, but is continually accelerated, as ſhe paſſes from the quarter to the new or full, and is retarded again by the like degrees in returning from the new and full to the next quarter. Here we conſider not ſo much the abſolute, as the apparent motion of the moon in reſpect to us.

10. THE principles of aſtronomy teach how to diſtinguiſh theſe two motions. Let S (in fig. 95.) repreſent the ſun, A the earth moving in its orbit BC, DEFG the moon's orbit, the place of the moon H. Suppoſe the earth to have moved from A to I. Becauſe it has been ſhewn, that the moon partakes of all the progreſſive motion of the earth; and likewiſe that the ſun attracts both the earth and moon equally, when they are at the ſame diſtance from it, or that the mean action of the ſun upon the moon is equal to its action

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[a] Newton's Princ. philoſ. Lib. III. prop. 22, 23.

upon the earth: we muſt therefore conſider the earth as carrying about with it the moon's orbit; ſo that when the earth is removed from A to I, the moon's orbit ſhall likewiſe be removed from its former ſituation into that denoted by KLMN. But now the earth being in I, if the moon were found in O, ſo that OI ſhould be parallel to HA, though the moon would really have moved from H to O, yet it would not have appeared to a ſpectator upon the earth to have moved at all, becauſe the earth has moved as much itſelf; ſo that the moon would ſtill appear in the ſame place with reſpect to the fixed ſtars. But if the moon be obſerved in P, it will then appear to have moved, its apparent motion being meaſured by the angle under OIP. And if the angle under PIS be leſs than the angle under HAS, the moon will have approached nearer to its conjunction with the ſun.

11. To come now to the explication of the mentioned inequality in the moon's motion: let S (in fig. 96.) repreſent the ſun, A the earth, BCDE the moon's orbit, C the place of the moon, when in the latter quarter. Here it will be nearly at the ſame diſtance from the ſun, as the earth is. In this caſe therefore they will both be equally attracted, the earth in the direction AS, and the moon in the direction CS. Whence as the earth in moving round the ſun is continually deſcending toward it, ſo the moon in this ſituation muſt in any equal portion of time deſcend as much; and therefore the poſition of the line AC in reſpect of AS, and the change, which the moon's motion produces in the angle under CAS, will not be altered by the ſun.

12. BUT now as ſoon as ever the moon is advanced from the quarter toward the new or conjunction, ſuppoſe to G, the action of the ſun upon it will have a different effect. Here, were the ſun's action upon the moon to be applied in the direction GH parallel to AS, if its action on the moon were equal to its action on the earth, no change would be wrought by the ſun on the apparent motion of the moon round the earth. But the moon receiving a greater impulſe in G than the earth receives in A, were the ſun to act in the direction GH, yet it would accelerate the deſcription of the ſpace DAG, and cauſe the angle under GAD to decreaſe faſter, than otherwiſe it would. The ſun's action will have this effect upon account of the obliquity of its direction to that, in which the earth attracts the moon. For the moon by this means is drawn by two forces oblique to each other, one drawing from G toward A, the other from G toward H, therefore the moon muſt neceſſarily be impelled toward D. Again, becauſe the ſun does not act in the direction GH parallel to SA, but in the direction GS oblique to it, the ſun's action on the moon will by reaſon of this obliquity farther contribute to the moon's acceleration. Suppoſe the earth in any ſhort ſpace of time would have moved from A to I, if not attracted by the ſun; the point I being in the ſtraight line CE, which touches the earth's orbit in A. Suppoſe the moon in the ſame time would have moved in her orbit from G to K, and beſides have partook of all the progreſſive motion of the earth. Then if KL be drawn parallel to AI, and taken equal to it, the moon, if not attracted by the ſun, would be found

in L. But the earth by the ſun's action is removed from I. Suppoſe it were moved down to M in the line IMN parallel to SA, and if the moon were attracted but as much, and in the ſame direction, as the earth is here ſuppoſed to be attracted, ſo as to have deſcended during the ſame time in the line LO, parallel alſo to AS, down as far as P, till LP were equal to IM, the angle under PMN would be equal to that under LIN, that is, the moon will appear advanced no farther forward, than if neither it nor the earth had been ſubject to the ſun's action. But this is upon the ſuppoſition, that the action of the ſun upon the moon and earth were equal; whereas the moon being acted upon more than the earth, did the ſun's action draw the moon in the line LO parallel to AS, it would draw it down ſo far as to make LP greater than IM; whereby the angle under PMN will be rendred leſs, than that under LIN. But moreover, as the ſun draws the earth in a direction oblique to IN, the earth will be found in its orbit ſomewhat ſhort of the point M; however the moon is attracted by the ſun ſtill more out of the line LO, than the earth is out of the line IN; therefore this obliquity of the ſun's action will yet farther diminiſh the angle under PMN.

13. Thus the moon at the point G receives an impulſe from the ſun, whereby her motion is accelerated. And the ſun producing this effect in every place between the quarter and the conjunction, the moon will move from the quarter with a motion continually more and more accelerated; and therefore by acquiring from time to time additional degrees

of velocity in its orbit, the ſpaces, which are deſcribed in equal times by the line drawn from the earth to the moon, will not be every where equal, but thoſe toward the conjunction will be greater, than thoſe toward the quarter. But now in the moon's paſſage from the conjunction D to the next quarter the ſun's action will again retard the moon, till at the next quarter in E it be reſtored to the firſt velocity, which it had in C.

14. AGAIN as the moon moves from E to the full or oppoſition to the ſun in B, it is again accelerated, the deficiency of the ſun's action upon the moon, from what it has upon the earth, producing here the ſame effect as before the exceſs of its action. Conſider the moon in Q moving from E towards B. Here if the moon were attracted by the ſun in a direction parallel to AS, yet being acted on leſs than the earth, as the earth deſcends toward the ſun, the moon will in ſome meaſure be left behind. Therefore QF being drawn parallel to SB, a ſpectator on the earth would ſee the moon move, as if attracted from the point Q in the direction QF with a degree of force equal to that, whereby the ſun's action on the moon falls ſhort of its action on the earth. But the obliquity of the ſun's action has alſo here an effect. In the time the earth would have moved from A to I without the influence of the ſun, let the moon have moved in its orbit from Q to R. Drawing therefore RT parallel to AI, and equal to the ſame, for the like reaſon as before, the moon by the motion of its orbit, if not at all attracted by the ſun, muſt be found in T; and therefore, if attracted in a direction parallel to SA, would

be in the line TV parallel to AS; ſuppoſe in W. But the moon in Q being farther off the ſun than the earth, it will be leſs attracted, that is, TW will be leſs than IM, and if the line SM be prolonged toward X, the angle under XMW will be leſs than that under XIT. Thus by the ſun's action the moon's paſſage from the quarter to the full would be accelerated, if the ſun were to act on the earth and moon in a direction parallel to AS: and the obliquity of the ſun's action will ſtill more increaſe this acceleration. For the action of the ſun on the moon is oblique to the line SA the whole time of the moon's paſſage from Q to T, and will carry the moon out of the line TV toward the earth. Here I ſuppoſe the time of the moon's paſſage from Q to T ſo ſhort, that it ſhall not paſs beyond the line SA. The earth alſo will come a little ſhort of the line IN, as was ſaid before. From theſe cauſes the angle under XMW will be ſtill farther leſſened.

15. THE moon in paſſing from the oppoſition B to the next quarter will be retarded again by the ſame degrees, as it is accelerated before its appulſe to the oppoſition. Becauſe this action of the ſun, which in the moon's paſſage from the quarter to the oppoſition cauſes it to be extraordinarily accelerated, and diminiſhes the angle, which meaſures its diſtance from the oppoſition; will make the moon ſlacken its pace afterwards, and retard the augmentation of the ſame angle in its paſſage from the oppoſition to the following quarter; that is, will prevent that angle from increaſing ſo faſt, as otherwiſe it would. And thus the moon, by the ſun's action upon it, is twice accelerated and twice reſtored to its firſt velocity, every

circuit it makes round the earth. This inequality of the moon's motion about the earth is called by aſtronomers its variation.

16. THE next effect of the ſun upon the moon is, that it gives the orbit of the moon in the quarters a greater degree of curvature, than it would receive from the action of the earth alone; and on the contrary in the conjunction and oppoſition the orbit is leſs inflected.

17. WHEN the moon is in conjunction with the ſun in the point D, the ſun attracting the moon more forcibly than it does the earth, the moon by that means is impelled leſs toward the earth, than otherwiſe it would be, and ſo the orbit is leſs incurvated; for the power, by which the moon is impelled toward the earth, being that, by which it is inflected from a rectilinear courſe, the leſs that power is, the leſs it will be inflected. Again, when the moon is in the oppoſition in B, farther removed from the ſun than the earth is; it follows then, though the earth and moon are both continually deſcending to the ſun, that is, are drawn by the ſun toward it ſelf out of the place they would otherwiſe move into, yet the moon deſcends with leſs velocity than the earth; inſomuch that the moon in any given ſpace of time from its paſſing the point of oppoſition will have leſs approached the earth, than otherwiſe it would have done, that is, its orbit in reſpect of the earth will approach nearer to a ſtraight line. In the laſt place, when the moon is in the quarter in F, and equally diſtant from the ſun as the earth, we obſerved before, that

the earth and moon would deſcend with equal pace toward the ſun, ſo as to make no change by that deſcent in the angle under FAS; but the length of the line FA muſt of neceſſity be ſhortned. Therefore the moon in moving from F toward the conjunction with the ſun will be impelled more toward the earth by the ſun's action, than it would have been by the earth alone, if neither the earth nor moon had been acted on by the ſun; ſo that by this additional impulſe the orbit is rendred more curve, than it would otherwiſe be. The ſame effect will alſo be produced in the other quarter.

18. Another effect of the ſun's action, conſequent upon this we have now explained, is, that though the moon undiſturbed by the ſun might move in a circle having the earth for its center; by the ſun's action, if the earth were to be in the very middle or center of the moon's orbit, yet the moon would be nearer the earth at the new and full, than in the quarters. In this probably will at firſt appear ſome difficulty, that the moon ſhould come neareſt to the earth, where it is leaſt attracted to it, and be fartheſt off when moſt attracted. Which yet will appear evidently to follow from that very cauſe, by conſidering what was laſt ſhewn, that the orbit of the moon in the conjunction and oppoſition is rendred leſs curve; for the leſs curve the orbit of the moon is, the leſs will the moon have deſcended from the place it would move into, without the action of the earth. Now if the moon were to move from any place without farther diſturbance from that action, ſince it would proceed in the line, which would touch its orbit in that place, it would

recede continually from the earth; and therefore if the power of the earth upon the moon, be ſufficient to retain it at the ſame diſtance, this diminution of that power will cauſe the diſtance to increaſe, though in a leſs degree. But on the other hand in the quarters, the moon, being preſſed more towards the earth than by the earth's ſingle action, will be made to approach it; ſo that in paſſing from the conjunction or oppoſition to the quarters the moon aſcends from the earth, and in paſſing from the quarters to the conjunction and oppoſition it deſcends again, becoming nearer in theſe laſt mentioned places than in the other.

19. ALL theſe forementioned inequalities are of different degrees, according as the ſun is more or leſs diſtant from the earth; greater when the earth is neareſt the ſun, and leſs when it is fartheſt off. For in the quarters, the nearer the moon is to the ſun, the greater is the addition to the earth's action upon it by the power of the ſun; and in the conjunction and oppoſition, the difference between the ſun's action upon the earth and upon the moon is likewiſe ſo much the greater.

20. THIS difference in the diſtance between the earth and the ſun produces a farther effect upon the moon's motion; cauſing the orbit to dilate when leſs remote from the ſun, and become greater, than when at a farther diſtance. For it is proved by Sir ISAAC NEWTON, that the action of the ſun, by which it diminiſhes the earth's power over the moon, in the conjunction or oppoſition, is about twice as

great, as the addition to the earth's action by the ſun in the quarters[a]; ſo that upon the whole, the power of the earth upon the moon is diminiſhed by the ſun, and therefore is moſt diminiſhed, when the action of the ſun is ſtrongeſt: but as the earth by its approach to the ſun has its influence leſſened, the moon being leſs attracted will gradually recede from the earth; and as the earth in its receſs from the ſun recovers by degrees its former power, the orbit of the moon muſt again contract. Two conſequences follow from hence: the moon will be moſt remote from the earth, when the earth is neareſt the ſun; and alſo will take up a longer time in performing its revolution through the dilated orbit, than through the more contracted.

21. These irregularities the ſun would produce in the moon, if the moon, without being acted on unequally by the ſun, would deſcribe a perfect circle about the earth, and in the plane of the earth's motion; but though neither of theſe ſuppoſitions obtain in the motion of the moon, yet the forementioned inequalities will take place, only with ſome difference in reſpect to the degree of them; but the moon by not moving in this manner is ſubject to ſome other inequalities alſo. For as the moon deſcribes, inſtead of a circle concentrical to the earth, an ellipſis, with the earth in one focus, that ellipſis will be ſubjected to various changes. It can neither preſerve conſtantly the ſame poſition, nor yet the ſame figure; and becauſe the plane of this ellipſis is not the ſame

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[a] Newton. Princ. Lib. I. prop. 66. coroll. 7.

with that of the earth's orbit, the ſituation of the plane, wherein the moon moves, will continually change; neither the line in which it interſects the plane of the earth's orbit, nor the inclination of the planes to each other, will remain for any time the ſame. All theſe alterations offer themſelves now to be explained.

22. I SHALL firſt conſider the changes which are made in the plane of the moon's orbit. The moon not moving in the ſame plane with the earth, the ſun is ſeldom in the plane of the moon's orbit, viz. only when the line made by the common interſection of the two planes, if produced, will paſs through the ſun, as is repreſented in fig. 97. where S denotes the ſun; T the earth; ATB the earth's orbit deſcribed upon the plane of this ſcheme; CDEF the moon's orbit, the part CDE being raiſed above, and the part CFE depreſſed under the plane of this ſcheme. Here the line CE, in which the plane of this ſcheme, that is, the plane of the earth's orbit and the plane of the moon's orbit interſect each other, being continued paſſes through the ſun in S. When this happens, the action of the ſun is directed in the plane of the moon's orbit, and cannot draw the moon out of this plane, as will evidently appear to any one that ſhall conſider the preſent ſcheme: for ſuppoſe the moon in G, and let a ſtraight line be drawn from G to S, the ſun draws the moon in the direction of this line from G toward S: but this line lies in the plane of the orbit; and if it be prolonged from S beyond G, the continuation of it will lie on the plane CDE; for the plane itſelf, if ſufficiently extended, will paſs through the ſun.

But in other caſes the obliquity of the ſun's action to the plane of the orbit will cauſe this plane continually to change.

23. SUPPOSE in the firſt place, the line, in which the two planes interſect each other, to be perpendicular to the line which joins the earth and ſun. Let T (in fig. 98, 99, 100, 101.) repreſent the earth; S the ſun; the plane of this ſcheme the plane of the earth's motion, in which both the ſun and earth are placed. Let AC be perpendicular to ST, which joins the earth and ſun; and let the line AC be that, in which the plane of the moon's orbit interſects the plane of the earth's motion. To the center T deſcribe in the plane of the earth's motion the circle ABCD. And in the plane of the moon's orbit deſcribe the circle AECF, one half of which AEC will be elevated above the plane of this ſcheme, the other half AFC as much depreſſed below it.

24. NOW ſuppoſe the moon to ſet forth from the point A (in fig. 98.) in the direction of the plane AEC. Here ſhe will be continually drawn out of this plane by the action of the ſun: for this plane AEC, if extended, will not paſs through the ſun, but above it; ſo that the ſun, by drawing the moon directly toward it ſelf, will force it continually more and more from that plane towards the plane of the earth's motion, in which it ſelf is; cauſing it to deſcribe the line AKGHI, which will be convex to the plane AEC, and concave to the plane of the earth's motion. But here this power of the ſun, which is ſaid to draw the moon toward the plane of the earth's motion, muſt be underſtood principally of ſo much only of the

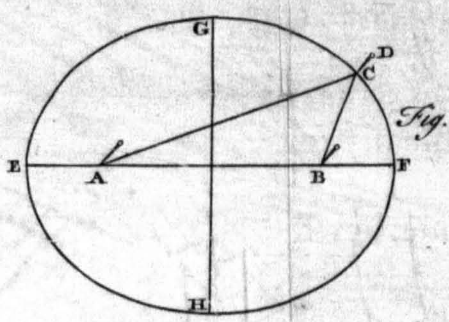

Fig. 91.

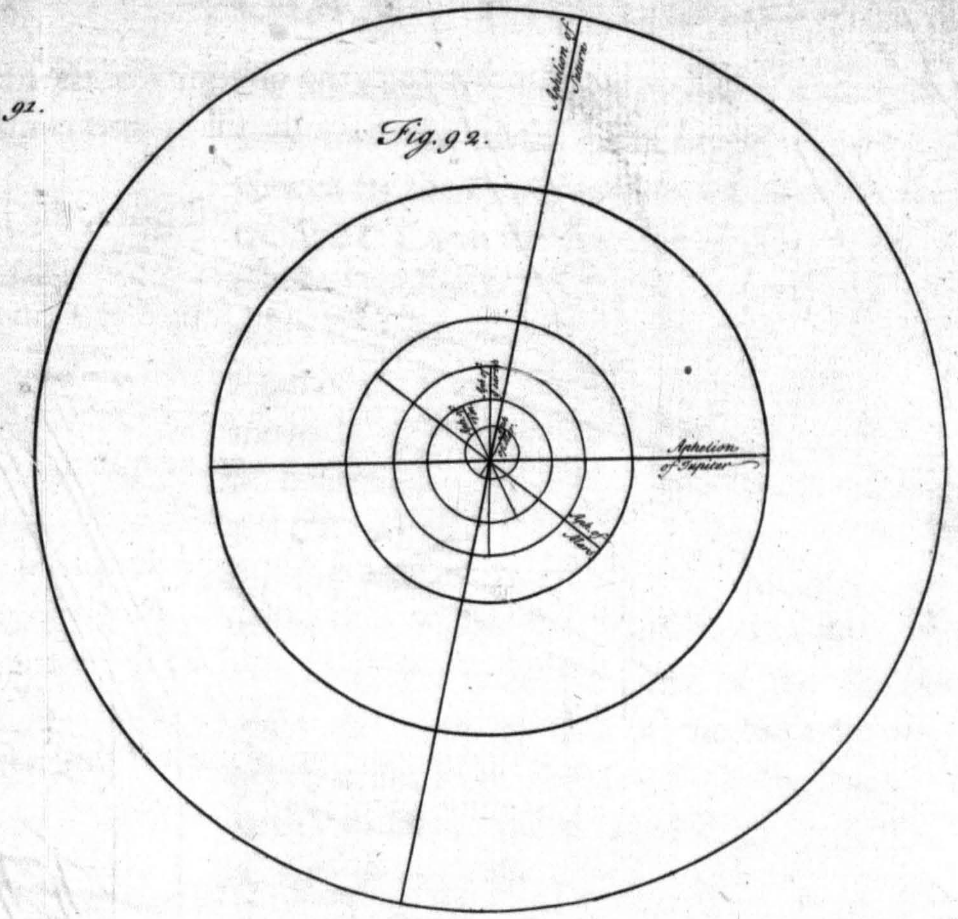

Fig. 92.

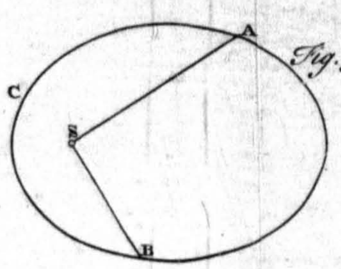

Fig. 93.

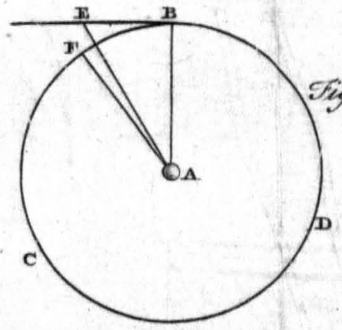

Fig. 94.

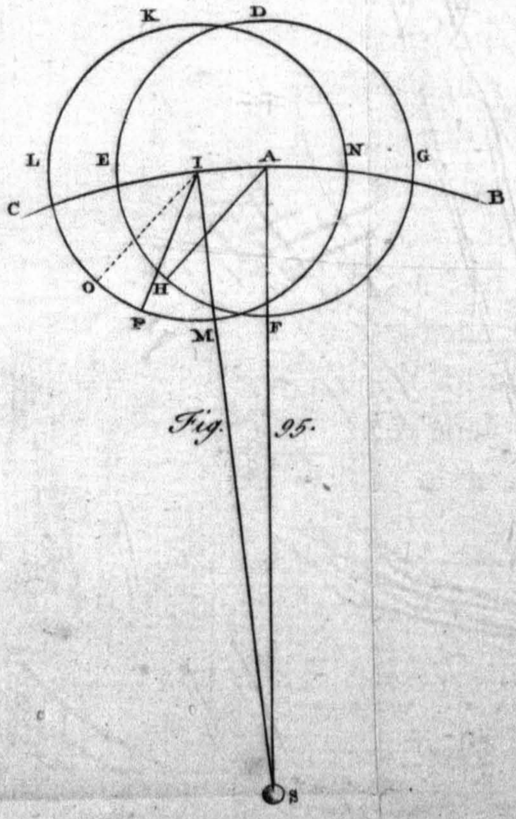

Fig. 95.

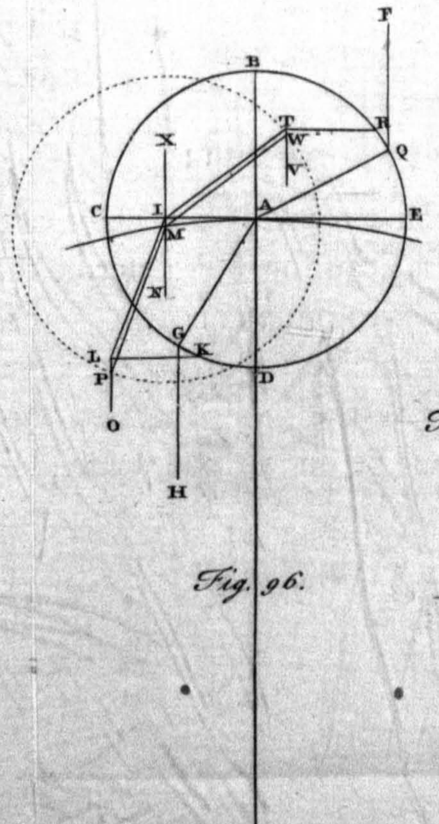

Fig. 96.

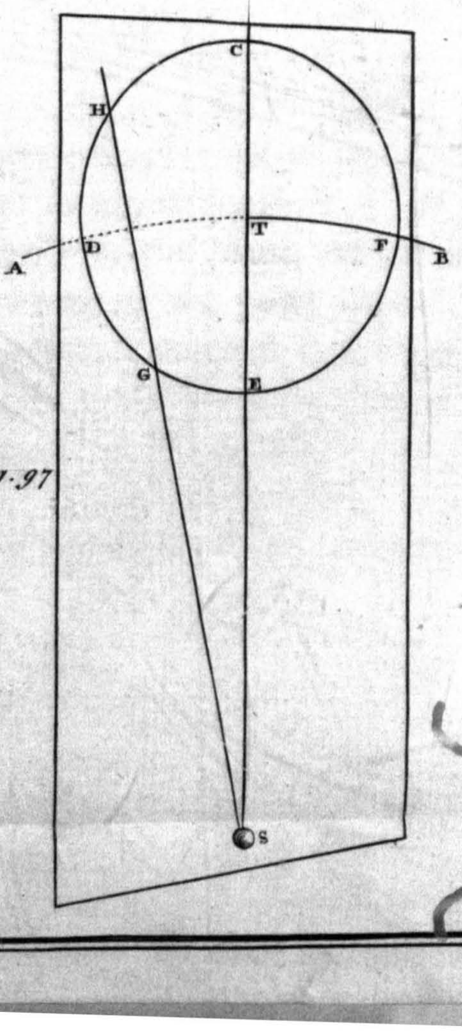

Fig. 97

the ſun's action upon the moon, as it exceeds the action of the ſame upon the earth. For ſuppoſe the preceding figure to be viewed by the eye, placed in the plane of that ſcheme, and in the line C T A on the ſide of A, the plane A B C D will appear as the ſtraight line D T B, (in fig. 102.) and the plane A E C F as another ſtraight line F E; and the curve line A K G H I under the form of the line T K G H I. Now it is plain, that the earth and moon being both attracted by the ſun, if the ſun's action upon both was equally ſtrong, the earth T, and with it the plane A E C F or line F T E in this ſcheme, would be carried toward the ſun with as great a pace as the moon, and therefore the moon not drawn out of it by the ſun's action, excepting only from the ſmall obliquity of the direction of this action upon the moon to that of the ſun's action upon the earth, which ariſes from the moon's being out of the plane of the earth's motion, and is not very conſiderable; but the action of the ſun upon the moon being greater than upon the earth, all the time the moon is nearer to the ſun than the earth is, it will be drawn from the plane A E C or the line T E by that exceſs, and made to deſcribe the curve line A G I or T G I. But it is the cuſtom of aſtronomers, inſtead of conſidering the moon as moving in ſuch a curve line, to refer its motion continually to the plane, which touches the true line wherein it moves, at the point where at any time the moon is. Thus when the moon is in the point A, its motion is conſidered as being in the plane A E C, in whoſe direction it then eſſaies to move; and when in the point K (in fig. 99.) its motion is referred to the plane, which paſſes through the earth, and touches the line A K G H I in the point K. Thus the